MULTIOBJECTIVE FORMULATION FOR NETWORK RESILIENCE: A TRADE-OFF BETWEEN VULNERABILITY AND RECOVERABILITY

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Abstract. The ubiquitous nature of infrastructure networks in today's society makes them a particularly important focus of preparedness planning, as their operation is essential for the many socioeconomic functions that rely upon them. Apart from that, many global disasters have prompted the need to study and plan for resilience. Despite previous work, which focus on after disruption partially, the work proposed here provides an initial multi-objective mathematical programming formulation based on reliability, vulnerability, and recoverability of the system to strengthen network resilience by emphasizing vulnerability and recoverability. The trade-off of investments made in both mitigation (vulnerability) and contingency (recoverability). Experimental results for both deterministic and stochastic conditions are presented, demonstrating the effectiveness and efficiency of the proposed model.

1 INTRODUCTION

The ubiquitous nature of infrastructure networks in today's society makes them a particularly important focus of preparedness planning, as their operation is essential for the many socioeconomic functions that rely upon them. No longer is it sufficient to focus on "prevention and protection" from the inevitability of disruptive events, potentially large-scale in nature. Recent natural disasters (e.g., hurricanes, earthquakes) have demonstrated an ability to overwhelm infrastructure networks regardless of the levels of prevention and protection. According the US National Academies of Science [2012], "One way to reduce the impacts of disasters on the nation and its communities is to invest in enhancing resilience […]."

The US government, through several agencies including the Department of Homeland Security (DHS), has increasingly emphasized resilience planning for critical infrastructure. Presidential Policy Directive 21 [Obama 2013] states that critical infrastructure "must be secure and able to withstand and rapidly recover from all hazards," where the combination of "withstanding" and "recovering" from disruptions constitutes resilience. *Resilience* has increasingly been seen in the literature [Hosseini et al. 2015, Park et al. 2013, Zolli and Healy 2012]. Ramirez-Marquez and co-authors offer a paradigm for system performance following a disruption, shown in Figure 1 [Henry and Ramirez-Marquez 2012, Barker et al. 2013, Pant et al. 2014, Baroud et al. 2014]. Network performance is quantified by a general performance measure $\varphi(t)$ (e.g., traffic flow or delay for a highway network). System resilience at time t is exhibited after a disruption, e^j, which affects the original system state. Based on this description, system resilience has been defined as a time-dependent and disruption-specific ratio $\mathrm{Recovery}(t)/\mathrm{Loss}(t_\mathrm{d})$.

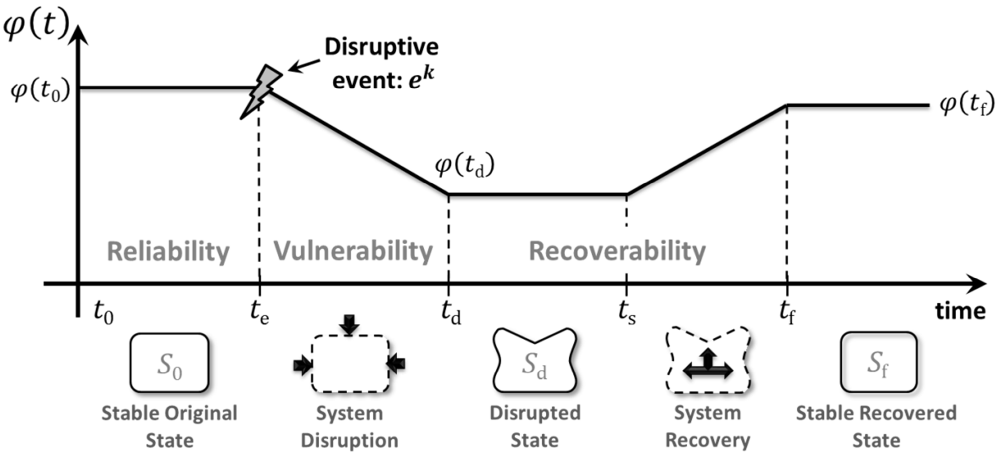

Figure 1. *System performance across system states.*

Figure 1 highlights two primary dimensions that resilient systems exhibit after a disruptive event: *vulnerability*, or an inability to maintain a desired performance level after a disruption, and *recoverability*, or an ability to recover timely. This paper addresses a multi-objective mathematical model for resilient networks which simultaneously considers (i) network vulnerability by reducing the impact initially experienced after a disruption, and (ii) network recoverability by finding the most effective ordering of the restoration of disrupted links.

The aim of this paper is to introduce an initial multiobjective mathematical model which (i) considers the proportional disruption in links, (ii) minimizes the vulnerability of network by assigning resources in presence of disaster, (iii) minimize the cost of unsatisfied demand (e.g. in transportation networks, the amount of traffic unable to reach a destination, or in electric power networks, the amount of electricity unable to be delivered from supplier nodes to customer nodes), (iv) minimizes the time of recovery by finding the best order of link to be recovered, (v) balances between the investments on vulnerability reduction and recoverability enhancement, and (vi) accounts for uncertainty in parameters of the formulation with stochastic optimization methods.

2 BRIEF BACKGROUND

Among the recent literature regarding network disruption and restoration, Lee and Wallace (2007) consider the importance of interdependent infrastructures, an example of five infrastructure such as communication, transportation, and power grids, in a network flow and represent a mathematical model to guide the system to restore after a disaster. The first attempts of resource selection and allocation to disrupted links in a network were made by Nurre et al. (2012), who develop a recovery process that determines which disrupted links to return to the problem, then optimally schedules their restoration based on the availability of work crews. Gong et al. [2013] study an interdependent supply chain network which integrates several underlying infrastructure networks (e.g., the power grid, communication, transportation), optimizing the cost of restoration and the performance of the multi-layer network. Shen [2013] develops a stochastic mixed integer model of the recovery of interdependent infrastructures under severe disruption. Baroud et al. [2014] develops a stochastic ordinal ranking approach to restoration of inland waterway networks based on two resilience-based importance measures from Barker et al. [2013].

3 PROBLEM DEFINITION AND STOCHASTIC OPTIMIZATION FORMULATION

The problem addresses a network $\mathcal{G}(N, L)$ consisting of a set of nodes N and a set of links L. This network includes three categories of nodes: N_{so} is the set of source nodes, N_{si} is the set of sink nodes, and N_t is the set of transmission nodes. The set D contains different disruptive scenarios that can affect the network, each of which reduces the operability of links by some percentage. In the event of a disruption, the network is serviced by a supplier of n_k types of resources (e.g., some work crews have specific equipment, some crews have a certain number of works). The rate of recovery per unit of time is λ. It is assumed that the rate of recovery for all links is equal, but the order in which a disrupted link is recovered affects the total restoration time of the network. The problem is described when a scenario disaster of type $d \in D$, occurs, and it disrupts the links proportionally from 0% to 100%. We must assign resources to disrupted links to lessen the percent of disruption. Then, the disrupted link are scheduled to be recovered one by one, and the order of recovery influences the total time of recovery.

Indices:

i, j Indices of nodes in the network, $i, j \epsilon N = \{1, \dots, n\}$,

$$N = N_{so} \cup N_{si} \cup N_t, N_{so} = \{1, \dots, n_{so}\}, N_t = \{n_{so} + 1, \dots, n_t\}, N_t = \{n_t + 1, \dots, n_{si}\}$$

k Index of resources available to be allocated to the disrupted links, $k \epsilon K = \{1, \dots, n_k\}$

d Set of scenario disasters which effect on network performance $d \epsilon D = \{1, \dots, n_d\}$

o Index of the order in which a link is recovered, $o \epsilon \{1, \dots, n_o\}$

θ Index of stochastic scenarios, $\theta \epsilon\ \Omega = \{1, \dots, n_\Omega\}$

Parameters:

n_o The number of disrupted links

P_{ijd} The proportional damage to link (i, j) when disaster type d happens

I_{ijkd} This is a factor whereby the vulnerability of link (i, j) is reduced when resource k is assigned to the link in the presence of disaster scenario type d

FP_{ij} The nominal capacity of link (i, j)

Co_{ijk} Cost of allocating resource k to (i, j)

Cc_k Cost of buying resource k

$Cf_{ijd\theta}$ The cost of performance reduction in link (i, j) after disaster under scenario d occurs in scenario θ

$h_{ijo\theta}$ The impact rate of link (i, j) recovery on network recovery when the link is recovered in the oth order

$Cr_{ijo\theta}$ The cost of recovery link (i, j) in order oth in scenario θ

$LS_{id\theta}$ The penalty cost for supply production loss in the presence of disaster scenario d and under stochastic scenario θ

$LD_{id\theta}$ The penalty cost for demand loss in the presence of disaster scenario d and under stochastic scenario θ

$R_{k\theta}$ The aggregation number of resource of type k in scenario θ

λ The flow recovery per time unit in scenario θ

T The time horizon for network recovery in scenario θ

B The total available budget in scenario θ

S_i The expected value of suppliers production at nodes $i \epsilon N_{so}$

D_i The demand expected to be satisfied at nodes $i \epsilon N_{si}$

π_θ The probability of stochastic scenario θ

Variables:

$Y_{ijk\theta}$ $\begin{cases} 1 & \text{if resource } k \text{ is allocated to link } (i.j) \text{ in scenario } \theta \\ 0 & \text{otherwise} \end{cases}$

$X_{ijo\theta}$ $\begin{cases} 1 & \text{if link } (i,j) \text{ is recovered in the } o\text{th order in scenario } \theta \\ 0 & \text{otherwise} \end{cases}$

$V_{k\theta}$ $\begin{cases} 1 & \text{if we use resource } k \text{ for reducing vulnerability in scenario } \theta \\ 0 & \text{otherwise} \end{cases}$

$F_{ijdk\theta}$ The flow between (i,j) during disaster type d with resource k assigned for scenario θ

$U_{id\theta}$ The slack variable related to supply node i during disaster d under scenario θ

$W_{id\theta}$ The slack variable related to demand node i during disaster d under scenario θ

Based on the above notation, we formulate the multiobjective problem, which balances vulnerability and recoverability, as follows. The first objective function, provided in Eq. (1), minimizes aspects of vulnerability by minimizing (i) the percentage of performance decrease after the disruption, (ii) the cost of contracting with resource suppliers, (iii) the cost of assigning a resource to a link, (iv) the cost related to the time when a supplier produces the level of services or the amount commodities which is less than the expected level of services or amount of commodities because of the disaster (the shortage in the amount of commodities or the level of the services a supplier provides), and (v) the cost of unmet demand in the network.

$$\begin{aligned} \min \sum_{\theta\in\Omega}\sum_{i\in N}\sum_{j\in N}\sum_{d\in D}\sum_{k\in K} \pi_\theta Cf_{ijd\theta} P_{ijd} I_{ijkd} Y_{ijk\theta} F_{ijdk\theta} + \sum_{\theta\in\Omega}\sum_{k\in K} \pi_\theta Cc_k V_{k\theta} - \sum_{\theta\in\Omega}\sum_{i\in N}\sum_{j\in N}\sum_{k\in K} \pi_\theta Co_{ijk} Y_{ijk\theta} \\ + \sum_{\theta\in\Omega}\sum_{d\in D}\sum_{i\in N} \pi_\theta LS_{id\theta}(S_i - U_{id\theta}) + \sum_{\theta\in\Omega}\sum_{d\in D}\sum_{i\in N} \pi_\theta LD_{id\theta}(D_i - W_{id\theta}) \\ + \sum_{\theta\in\Omega}\sum_{j\in N}\sum_{i\in N}\sum_{k\in K}\sum_{d\in D}\sum_{o\in O} \pi_\theta Cr_{ijo\theta}\big(P_{ijd\theta} I_{ijk\theta}\big) Y_{ijk\theta} F_{ij\theta} \end{aligned} \tag{1}$$

The second objective function, provided in Eq. (2), maximizes the enhancement in recovery time for link (i,j) recovered in the oth order.

$$\max \sum_{\theta\in\Omega}\sum_{j\in N}\sum_{i\in N}\sum_{o\in O}\sum_{d\in D}\sum_{k\in K} \frac{(\pi_\theta h_{ijo\theta}(1 - P_{ijl\theta} I_{ijk\theta}) Y_{ijk\theta} F_{ijdk\theta})}{\lambda} X_{ijo\theta} \tag{2}$$

Constraint (3) ensures that a resource must be assigned to a link that is vulnerable to a disruption and potentially becomes disrupted when a disruptive event occurs. Constraint (4) matches repaired links with the appropriate number of resources, and Constraint (5) ensures that only disrupted nodes are repaired. Constraint (6) shows the resource usage limitation.

$$\sum_{k \in K} Y_{ijk\theta} \geq P_{ijd\theta} \qquad \forall i,j \in N, \forall d \in D, \forall \theta \in \Omega \tag{3}$$

$$\sum_{j \in N} \sum_{i \in N} Y_{ijk\theta} \geq V_{k\theta} \qquad \forall k \in K, \forall \theta \in \Omega \tag{4}$$

$$\sum_{k \in K} V_{k\theta} \geq P_{ijd\theta} \qquad \forall i,j \in N, \forall d \in D, \forall \theta \in \Omega \tag{5}$$

$$\sum_{j \in N} \sum_{i \in N} Y_{ijk\theta} \leq R_{k\theta} \qquad \forall k \in K, \forall \theta \in \Omega \tag{6}$$

Constraints (7)-(9) calculate the amount of commodities and services that suppliers provide and target demands, as well as make sure that transmission nodes do not increase or decrease the amount of flow.

$$\sum_{j \in N} (1 - P_{ijd\theta} I_{ijkd\theta)} Y_{ijk\theta} F_{ijdk\theta} - \sum_{j \in N} (1 - P_{ijd\theta} I_{ijkd\theta)} Y_{ijk\theta} F_{jidk\theta} = U_{idk\theta} \\ \forall i \in N_{so}, \forall k \in K, \forall d \in D, \forall \theta \in \Omega \tag{7}$$

$$\sum_{j \in N} (1 - P_{ijd\theta} I_{ijkd\theta)} Y_{ijk\theta} F_{ijdk\theta} - \sum_{j \in N} (1 - P_{ijd\theta} I_{ijkd\theta)} Y_{ijk\theta} F_{jidk\theta} = W_{idk\theta} \\ \forall i \in N_{si}, \forall k \in K, \forall d \in D, \forall \theta \in \Omega \tag{8}$$

$$\sum_{j \in N} (1 - P_{ijd\theta} I_{ijkd\theta)} Y_{ijk\theta} F_{ijdk\theta} - \sum_{j \in N} (1 - P_{ijd\theta} I_{ijkd\theta)} Y_{ijk\theta} F_{jidk\theta} = 0 \\ \forall i \in N_{t}, \forall k \in K, \forall d \in D, \forall \theta \in \Omega \tag{9}$$

Constraints (10) and (11) represent supply and demand capacity limitations, respectively.

$$U_{idk\theta} \leq S_{i\theta} \qquad \forall i \in N_{so}, \forall k \in K, \forall d \in D, \forall \theta \in \Omega \tag{10}$$

$$W_{idk\theta} \leq D_{i\theta} \qquad \forall i \in N_{si}, \forall k \in K, \forall d \in D, \forall \theta \in \Omega \tag{11}$$

Constraint (12) requires that if link (i,j) is disrupted, at least one resource should be assign to it to reduce its vulnerability. Constraints (13) if an edge is disrupted by a disaster it should be recovered

$$X_{ijo\theta} \leq \sum_{k \in K} Y_{ijk\theta} \qquad \forall (i,j) \in N, \forall k \in K, o \in O, \forall \theta \in \Omega \tag{12}$$

$$\sum_{o\in O} X_{ijo\theta} = 1 \qquad \forall (i,j) \in N, \forall k \in K, \forall \theta \in \Omega \tag{13}$$

Constraint (14) confirms flow capacity, constraint (15) represents the limitation on the recovery time horizon, and constraint (16) represents the limitation on the budget.

$$F_{ijkd\theta} \leq FP_{ij\theta} \qquad \forall (i,j) \in N, \forall k \in K, o \in O, \forall \theta \in \Omega \tag{14}$$

$$\begin{aligned}\max_{o=1}&\left\{\sum_{i\in N}\sum_{j\in N}\sum_{k\in K}\sum_{d\in D}\left(\left(P_{ijdl\theta}I_{ijk\theta}\right)Y_{ijk\theta}F_{ijdlk\theta}\Big/_{\lambda}\right)X_{ijo\theta}\right\}+\cdots\\&+\max_{o=p}\left\{\sum_{i\in N}\sum_{j\in N}\sum_{k\in K}\sum_{d\in D}\left(\left(P_{ijdl\theta}I_{ijk\theta}\right)Y_{ijk\theta}F_{ijdlk\theta}\Big/_{\lambda}\right)X_{ijo\theta}\right\}\leq T\end{aligned} \qquad \forall \theta \in \Omega \tag{15}$$

$$\sum_{j\in N}\sum_{i\in N}\sum_{k\in K} Co_{ijk\theta}Y_{ijk\theta} + \sum_{j\in N}\sum_{i\in N}\sum_{o\in O} Cr_{ijo\theta}X_{ijo\theta} \leq T \qquad \forall \theta \in \Omega \tag{16}$$

$$\begin{matrix} Y_{ijk\theta}, X_{ijo\theta}, V_{k\theta} \epsilon \{0,1\} \\ F_{ijdk\theta}, U_{id\theta}, W_{id\theta} \geq 0 \end{matrix} \qquad \forall (i,j) \in N, \forall k \in K, o \in O, \forall \theta \in \Omega \tag{17}$$

4 COMPUTATIONAL RESULTS

To develop stochastic optimization in the model, in vulnerability section, the cost of performance reduction in each link, the penalty costs, and the number of available resources are considered as uncertain parameters, and in the recovery section, the recovery cost and the rate of performance efficiency, when links are recovered in scheduled pattern, are considered as uncertain parameters. In order to model the problem under uncertainty the stochastic scenario based optimization is used in this paper. Let Ω be the set of all possible scenario and θ is a particular scenario. If π_θ denotes the probability of scenarioθ, because θ is a finite number (number of scenario is four,$\theta \in \{1,2,3,4\}$) the expected value function becomes a summation on θ. We consider four scenarios $\theta \in \{1,2,3,4\}$ which randomly are chosen to have a specific probability of occurrence. Each scenario has a determined set of parameters which lead to specified set of output for the model [Pishvaee et al. 2008].

Increasing the total number of scenarios can lead to a significant increase in the computation time [El-seyed et al. 2010]. Consequently, to limit the number of scenarios, a fuzzy clustering-based method presented by Pishvaee et al. [2008] was used to obtain a reasonable number of scenarios, in this case four scenarios, for a test problem shown in Table 1.

Table 1. *The range of parameters for four different scenarios.*

Scenario (θ)	Scenario probability (π_θ)	$Cf_{ijd\theta}$	$h_{ijo\theta}$	$Cr_{ijo\theta}$	$LS_{id\theta}$	$LD_{id\theta}$
1	0.4	~Unif[10,100]	~Unif[1,2]	~Unif[250,370]	~Unif[100,120]	~Unif[110,190]
2	0.3	~Unif[50,111]	~Unif[1.5,1.5]	~Unif[200,390]	~Unif[100,120]	~Unif[150,190]
3	0.2	~Unif[25,150]	~Unif[1,1.8]	~Unif[150,450]	~Unif[100,120]	~Unif[130,200]

4	0.1	~Unif[30,80]	~Unif[1.1,1.7]	~Unif[300,570]	~Unif[100,120]	~Unif[110,155]

For comparing the deterministic and the stochastic one is used as the nominal data for the deterministic model. Table 2 shows the experimental results of solving both deterministic and stochastic models and, moreover, depicts the reality that the stochastic objective function is more than that the deterministic objective function as the result of considering worst case situations, and the higher level of complexity as the result of having more constraints and decision variable having one more dimension.

Table 2. *Computational result under nominal data.*

Objective function	Optimal value of objective function		Number of variables		Number of constraints	
	Deterministic	Stochastic	Deterministic	Stochastic	Deterministic	Stochastic
	2889593	10543970				
Vulnerability(Obj.1)	2892070	10548584				
	2897416	10573195				
	8692506	29221055	240	957	357	1428
Recovery(Obj.2)	4346253	14610528				
	869250	2922106				

Figures 2 and 3 depict the Pareto optimal solution for both deterministic and stochastic models, respectively. As it is seen in Figure 3, the value of the vulnerability and recovery objectives have higher values than the objective functions for the deterministic model, suggesting that under different scenario with different probabilities, provides the model with the robustness whereby the model can tolerate abrupt changes in the parameters.

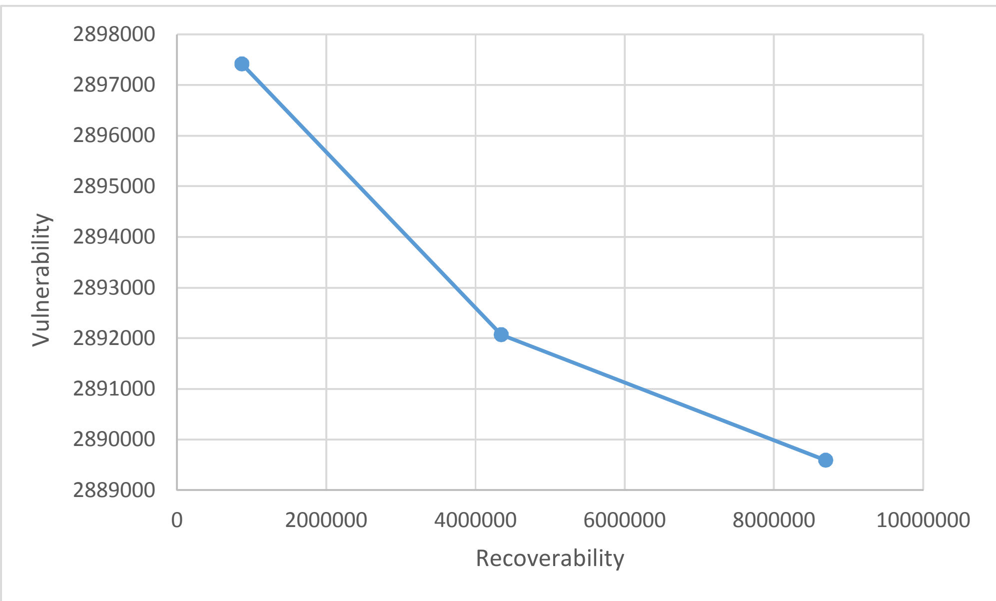

Figure 2. *The Pareto optimal solution for deterministic model.*

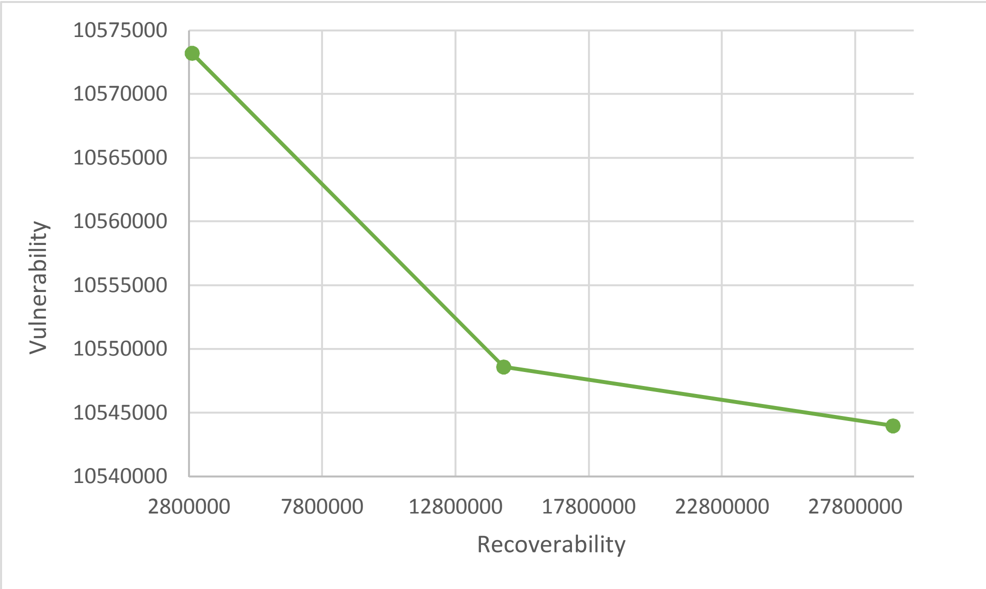

Figure 3. *The Pareto-optimal solution for stochastic model.*

5 CONCLUSIONS

This work provides a first step in developing an optimization framework for resource allocation to enhance network resilience wherein vulnerability and recoverability are treated as competing objectives. We offer a stochastic approach considering the parameter, which are inconstant in the real world, and make them flow easily in the distribution they follow instead of considering them as unique parameters. The results, both for deterministic and stochastic, depict the contradiction between two objective functions and furthermore, the higher results produced by the stochastic model and its ability to produce output under different scenarios shows the robustness of the stochastic model under the uncertain situation. Future work includes a more detailed analysis of multiple scenarios, as well as a data-driven study of real infrastructure network disruptions.

6 REFERENCES

Murray, A.T. and T.H. Grubesic. 2012. Critical infrastructure protection: The vulnerability conundrum. *Telematics and Informatics*, **29**(1): 56-65.

Barker, K., J.E. Ramirez-Marquez, and C.M. Rocco. 2013. Resilience-based network component importance measures. *Reliability Engineering and System Safety,* **117**: 89-97.

Barker, K., and H. Baroud. 2014. Proportional hazards models of infrastructure system recovery. *Reliability Engineering and System Safety*, **124**: 201-206.

Baroud, H., K. Barker, J.E. Ramirez-Marquez, and C.M. Rocco. 2014. Importance measures for inland waterway network resilience. *Transportation Research Part E*, **62**: 55-67.

Devanandham, H. and J.E. Ramirez-Marquez. 2012. Generic metrics and quantitative approaches for system resilience as a function of time. *Reliability Engineering and System Safety*, **99**: 114-122.

El-Sayed, M., N. Afia, and A. El-Kharbotly. 2010. A stochastic model for forward reverse logistics network design under risk. *Computers and Industrial Engineering*, **58**: 423-31.

Gong, J., J.E. Mitchell, A. Krishnamurthy, and W.A. Wallace. 2014. An interdependent layered network model for a resilient supply chain. *Omega*, **46**: 104-116.

Hosseini, S., K. Barker, and J.E. Ramirez-Marquez. 2015. A Review of Definitions and Measures of System Resilience. In revision in *Reliability Engineering and System Safety*.

Lee, E., J.E. Mitchell, W.A. Wallace. 2007. Restoration of services in interdependent infrastructure systems: a network flows approach. *IEEE Transactions on Systems, Man, and Cybernetics, Part C*, **37**(6): 1303-1317.

Nurre, S.G., B. Cavdaroglu, J.E. Mitchell, T.C. Sharkey, and W.A. Wallace. 2012. Restoring infrastructure systems: An integrated network design and scheduling (INDS) problem. *European Journal of Operational Research*, **223**(3): 794-806.

Pant, R., K. Barker, J.E. Ramirez-Marquez, C.M. Rocco. 2014. Stochastic measures of resilience and their application to container terminals. *Computers and Industrial Engineering,* **70**: 183-194.

Pant, R., K. Barker, and C.W. Zobel. 2014. Static and dynamic metrics of economic resilience for interdependent infrastructure and industry sectors. *Reliability Engineering and System Safety*, **125**: 92-102.

Park, J., T.P. Seager, P.S.C. Rao, M. Convertino, and I. Linkov. 2013. Integrating Risk and Resilience Approaches to Catastrophe Management in Engineering Systems. *Risk Analysis*, **33**(3): 356-367.

Pishvaee, M.S., M. Fathi, and F. Jolai. 2008. A fuzzy clustering-based method for scenario analysis in strategic planning: the case of an Asian pharmaceutical company. *South African Journal of Business Management*, **39**: 15-25.

Shen, S. 2013. Optimizing designs and operations of a single network or multiple interdependent infrastructures under stochastic arc disruption. *Computers and Operations Research*, **40**(11): 2677-2688.

US National Academies of Science. 2012. *Disaster Resilience: A National Imperative*. Washington, DC: The National Academies Press.

Zolli, A. and A.M. Healy. 2012. *Resilience: Why Things Bounce Back*. New York, NY: Free Press.